7. Summary

- Convexity, algebra, and duality.

- A synergy of algebraic and numerical methods.

- Development of relationship between duality, algebra, and geometry.

- Constructive methodology for practically relevant questions.

- A useful *computational framework* for semialgebraic problems.

- Numerous *applications*, same new basic *tools*.

- Sum of squares methods provide easily computable, certified solutions.

- A broad generalization of known successful techniques.

- Tradeoff between accuracy vs. computation time.

Certificates

Degree \ Field	Complex	Real
Linear	*Range/Kernel* Linear Algebra	*Farkas Lemma* Linear Programming
Polynomial	*Nullstellensatz* Bounded degree: LP Groebner bases	*Positivstellensatz* Bounded degree: SDP

Conclusions

Methodological aspects

- Constructive and algebraic aspects of duality.

- The lifting/relaxation/duality general scheme

- Hierarchies of relaxations, certificate complexity.

- Unification of many particular cases.

Concrete applications

- Continuous and combinatorial optimization

- Lyapunov and density function computation

- Many others: quantum entanglement, geometric theorem proving, etc.

Open research topics

- Practicalities. Implementation, numerical conditioning, etc.

- How big are the problems that we can solve?

- General theory for *a priori* or approximation guarantees?

- How can we exploit the problem structure for more efficient solutions?

- What are the computational complexity implications?

Lots of things to do!