13. Summary

- Convexity, algebra, and duality.
- A synergy of algebraic and numerical methods.
- Development of relationship between duality, algebra, and geometry.
- Constructive methodology for practically relevant questions.
- A useful *computational framework* for semialgebraic problems.
- Numerous *applications*, same new basic *tools*.
- Sum of squares methods provide easily computable, certified solutions.
- A broad generalization of known successful techniques.
- Tradeoff between accuracy vs. computation time.

Certificates

Degree \ Field	Complex	Real
Linear	*Range/Kernel* Linear Algebra	*Farkas Lemma* Linear Programming
Polynomial	*Nullstellensatz* Bounded degree: LP Groebner bases	*Positivstellensatz* Bounded degree: SDP

Conclusions

Methodological aspects

- Constructive and algebraic aspects of duality.
- The lifting/relaxation/duality general scheme
- Hierarchies of relaxations, certificate complexity.
- Completely algorithmic methodology.
- Unification of many particular cases.

Concrete applications

- Continuous and combinatorial optimization
- Lyapunov and density function computation
- Many others: quantum entanglement, geometric theorem proving, etc.

Open research topics

- Practicalities. Implementation, numerical conditioning, etc.
- How big are the problems that we can solve?
- General theory for *a priori* or approximation guarantees?
- How can we exploit the problem structure for more efficient solutions?
- What are the computational complexity implications?

Lots of things to do!