Welcome to 6.S979

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- Join Piazza !
- See syllabus on Canvas

QM $\Longleftrightarrow$ classical probability

Outline

1) Physics (Bell's thm, CHSH, ...)
2) Math (Quantum correlations)
3) CS (MIP*, cryptography)

1) Quantum basics

- State
- Measurements / observations
- Transformations

Quantum state $|\psi\rangle \in \vec{\mathcal{H}}$

Ex. qubit $\mathbb{C}^2$ $|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

Measurements:

Projective measurement $\{\Pi_a\}$

- $\sum_a \Pi_a = I$
- $\Pi_a \Pi_b = 0$ for $a \neq b$

$$\Pr[a] = \underbrace{\langle\psi|}_{\text{bra}} \Pi_a \underbrace{|\psi\rangle}_{\text{ket}}$$

braket

$$\begin{pmatrix} \alpha^* & \beta^* \end{pmatrix} \qquad \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$|\psi\rangle \to \gamma|\psi\rangle \quad |\gamma| = 1$

makes no difference to prob.

Collapse of state:

If you measure and get outcome a

$$|\psi\rangle \longrightarrow \frac{\Pi_a|\psi\rangle}{\|\Pi_a|\psi\rangle\|}$$

Example:

Qubit $\mathbb{C}^2$

Standard basis is $|0\rangle, |1\rangle$

Z-basis

Measurement

$\Pi_0^Z = |0\rangle\langle 0|$ $\Pi_1^Z = |1\rangle\langle 1|$

$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

X-basis

$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

X-basis measurement

$\Pi_+^X = |+\rangle\langle +|$, $\Pi_-^X = |-\rangle\langle -|$

$|\psi\rangle = |0\rangle$

$$\Pr_Z[0] = \langle 0 | \Pi_0^Z | 0 \rangle$$
$$= \langle 0 | \; | 0 \rangle \langle 0 | \; | 0 \rangle = 1$$

$$\Pr_Z[1] = \langle 0 | \Pi_1^Z | 0 \rangle = 0$$

$$\Pr_X[+] = \langle 0 | \; \Pi_0^X \; | 0 \rangle$$
$$= \langle 0 | \; |+\rangle\langle +| \; | 0 \rangle$$
$$= 1/2$$

$$\Pr_X[-] = 1/2$$

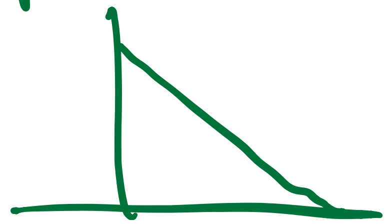
Post-measurement state

$$\frac{\Pi_- |0\rangle}{\| \Pi_- |0\rangle \|} = \frac{\frac{1}{\sqrt{2}} |-\rangle}{\| \frac{1}{\sqrt{2}} |-\rangle \|}$$
$$= |-\rangle$$

Transformations: Unitary matrices

Classical prob:

State: $v \in$ prob. simplex



Measurement: collection $\{u_a\}$ indicator

Coin: $v = (p_{heads}, p_{tails})$

Measure: $u_{heads} = (1, 0)$, $u_{tails} = (0, 1)$

$\Pr[a] = u_a \cdot v$

Transformations: stochastic matrices

Subsystems & Entanglement

QM: System A $\mathcal{H}_A$

System B $\mathcal{H}_B$

Systems A & B $\mathcal{H}_A \otimes \mathcal{H}_B$

E.g. 2 qubits

$\mathbb{C}^2 \otimes \mathbb{C}^2$

Basis: $|0\rangle \otimes |0\rangle$, $|0\rangle \otimes |1\rangle$, $|1\rangle \otimes |0\rangle$, $|1\rangle \otimes |1\rangle$

$|+\rangle \otimes |+\rangle$, $|+\rangle \otimes |-\rangle$, $|-\rangle \otimes |+\rangle$, $|-\rangle \otimes |-\rangle$

"Measure the first qubit"

$$\Pi_a \otimes I$$

$$|0\rangle\langle 0| \otimes I, \quad |1\rangle\langle 1| \otimes I$$

EPR pair:

$|0\rangle \otimes |0\rangle$ $\quad$ $|1\rangle \otimes |1\rangle$

$$|EPR\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

"product state"

$\neq |\psi_A\rangle \otimes |\psi_B\rangle$

"entangled"

Q: What if we measure 1st qubit in Z-basis?

$$Pr_Z[0] = \langle EPR| \underbrace{|0\rangle\langle 0| \otimes I}\, |EPR\rangle$$

$$= \langle EPR| \frac{1}{\sqrt{2}}|00\rangle$$

$$= \frac{1}{2} \longrightarrow$$ post-measurement state $|00\rangle$

$$Pr_Z[1] = 1/2$$

$\longrightarrow$ If qubit 1 gives 0, then qubit 2 will give 0 as well

Q: What about X basis?

$$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

$$|1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$$

$$|00\rangle = |0\rangle \otimes |0\rangle$$

$$= \frac{1}{2}\left(|++\rangle \pm |+-\rangle \pm |-+\rangle + |--\rangle\right) \Leftarrow$$

$$|11\rangle = |1\rangle \otimes |1\rangle$$
$$= \frac{1}{2}\left(|++\rangle - |+-\rangle - |-+\rangle + |--\rangle\right)$$

$$|EPR\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$
$$= \frac{1}{\sqrt{2}}\left(|++\rangle + |--\rangle\right)$$

Incompatible measurements:

Π_0 Π_1

$|\psi\rangle$ has a predictable outcome iff it is an eigenstate of Π_0 and Π_1

Q: When can 2 measurements be done simultaneously?

A: Only if simultaneously diagonalizable

Ex of non-compatible measmnts

$|\psi\rangle = |0\rangle$

first measure Z, then X

$\Pr[0] = 1$

$\Pr[+] = 1/2$
$\Pr[-] = 1/2$

first measure X, then Z

$\Pr[+] = 1/2 \xrightarrow{|+\rangle} \Pr[0] = 1/2$
$\Pr[-] = 1/2 \longrightarrow \Pr[1] = 1/2$

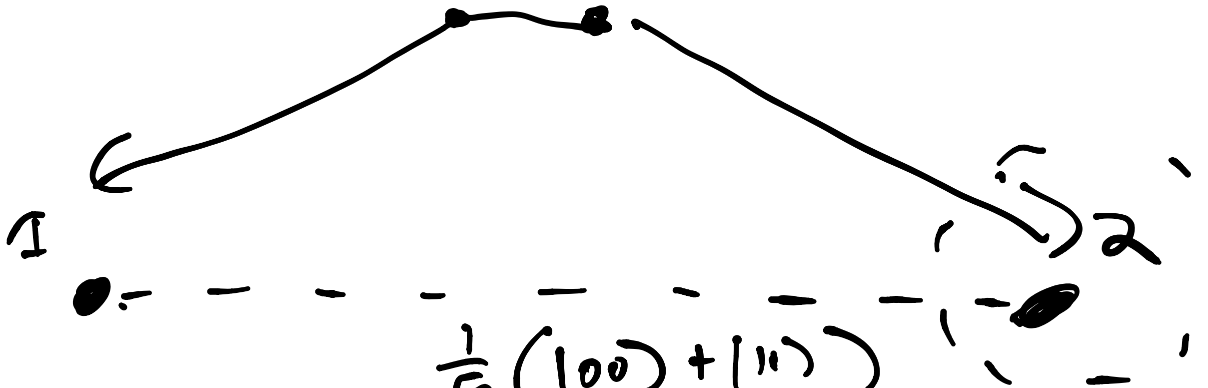
Heisenberg:

There is no state w/ predictable X & Z outcomes

$\mathrm{var}(X) \cdot \mathrm{var}(Z) \geq \ldots$

Einstein Podolsky Rosen paradox:

$$|EPR\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



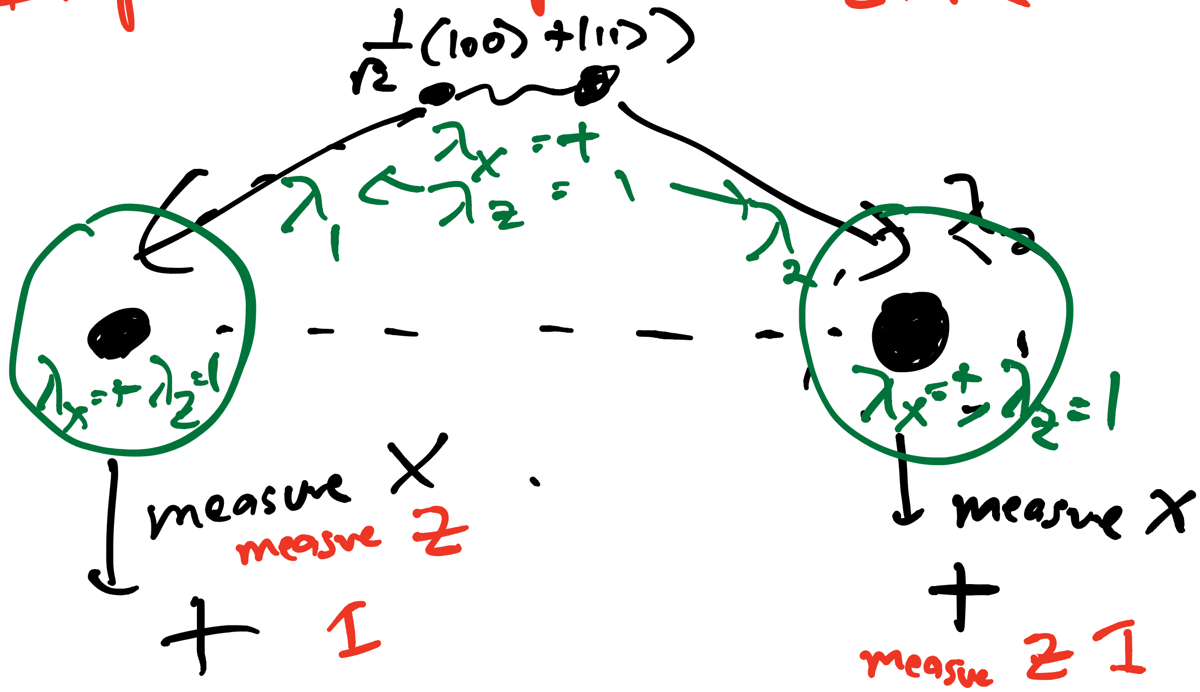
$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\text{Outcome}_1^X = + \implies \text{Outcome}_2^X = +$$

$$\text{Outcome}_1^Z = 0 \implies \text{Outcome}_2^Z = 0$$

__Conclusion:__ There is extra, hidden info in nature not encoded in q. state

Implicit assumption in EPR



Bell '64

QM is inconsistent with local hidden vars. Simple experiment to rule out LHV.